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The quantitative model
of an open-end investment fund

Abstract of the doctoral thesis

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Motivation of the thesis

Over the last three decades, the global financial market has undergone significant transformations, which have resulted in the institutionalization of investing. As a result one of the most important financial institutions that today hold financial assets of retail investors are open-end investment funds, named also mutual funds. The development and potential of this market is huge and connected to the fact that today collecting capital for retirement has become a necessity and an obligation.

Since the times when the modern portfolio theory by Markowitz (1952, 1959), the capital asset pricing model by Sharpe (1964), efficient market hypothesis by Fama (1970), or prospect theory by Kahneman and Tversky (1979) were introduced, researchers have studied the topic of open-end investment fund performance and its determinants that influence the allocation decisions of retail investors very deeply. There are thousands of studies on fund performance measurement and its relation to the fund net flows or different fund attributes, such as charges that lower the net return of the funds (the most recognized are among others (Berk & Green, 2004; Carhart, 1997; Fama & French, 2010; Gil-Bazo & Ruiz-Verdu, 2009; Jensen, 1969; Sharpe, 1994)). This relation is studied mostly from the demand side, i.e. from the side of investors participating in these funds. There has been little attention paid to the supply side - investment companies and their managers who optimize their decisions by operating in a competitive surrounding. This task is definitely difficult due to the significantly limited amount of commonly available data. As a consequence, the possibility of comparing own products with competitors is significantly limited and subject to high uncertainty. Additionally, in the literature, the processes that make up the operational functioning of mutual funds are usually interpreted selectively and independently, in a highly simplified manner. As a result, the description of fund operations does not use all available data in a way that fully exhausts their information value.

The perceived research gap, as well as the author's professional experience as a data analyst in an investment fund company, have become a motivation to build a comprehensive and detailed model that will quantitatively and thus objectively describe the functioning of open-end investment funds, including their interactions with the surrounding. In this doctoral dissertation, the author undertakes the task of developing a mutual fund functioning model, which will allow for a reliable analysis of any selected parameters (e.g.

cash flows, performance or costs). Moreover, thanks to the holistic approach, a standardized description method will also be proposed for the uncertainty resulting either from the adopted method or the unavailability of some data.

Research aims and questions

The main aim of the thesis is to construct a detailed and comprehensive quantitative model of three types of operations of an open-end investment fund: including cash flows, performance and costs. The model takes the form of the mathematical formulas describing the fund's operations mentioned. In order to build and test the model the detailed aims of the dissertation were realized:

- embedding the model theoretically in the context of an organization as a system and developing an approach to describing its functioning, as well as its accompanying uncertainty;
- review of the scientific literature on the key aspects and challenges of the operations of open-end investment funds, including quantitative approaches to measuring capital flows and performance;
- presenting the characteristics of the investment fund market in Poland as an example of a market with high dynamics of changes in the functioning of CIF;
- construction of a quantitative model of the operation of an open-end investment fund;
- simulations and analyzes showing the scope of application of the constructed model.

The author asks the following research questions:

- How to accurately estimate the value of the fund's net flows representing the decisions of its participants?
- How to accurately and unequivocally describe the fund performance before fees in order to assess and compare the effectiveness of the fund's investment portfolio managers?
- How to comprehensively analyze fund fees and costs, in particular other than the management fee?

The Author answers those questions by realizing the aims of the study.

Structure of the thesis

The thesis is divided into five chapters. The first three provide a substantive introduction to the areas that form the basis of this study, while the last two chapters are connected to the presentation of the model of functioning of a mutual fund and its accuracy simulations. The structure of the PhD thesis reflect realization of the detailed aims of the study. It is presented in Table 1.

Table 1. Structure of the thesis as a reflection of detailed aims of this study

Detailed aim	Realization description in chapters
Theoretically embedding the model in the context of an organization as a system and developing an approach to describing its functioning, as well as the uncertainty connected with measuring the aspects of this functioning.	The goal was realized in Chapter 1. The reference to the approach to organization as a system in management sciences was presented and this concept was extended to include a quantitative description in finance. Based on this, the approach to measuring the functioning of an organization as a system and a description of its uncertainty are presented – the methods were drawn from a standardized approach and international standards in life sciences. The discussion is summed up by considerations referring to the philosophy of science and the description of the system in such approach.
Review of the scientific research on the key aspects and challenges of the operation of an open-end investment funds, including quantitative approaches to measuring capital flows and performance.	The goal was realized in Chapter 2. A comprehensive and exhaustive literature review was the basis for the construction of the model. The analysis covers the key aspects of functioning an open-ended investment fund, i.e. investment results (performance) and net flows. An important observation was the quite selective approach to establishing formulas for measures that describe the functioning of the fund. The key challenges for the functioning of the funds were also identified, for which the author considered primarily fees and costs, but also investment limits and liquidity.
Presentation of the	The goal was realized in Chapter 3. Analyzing a specific

<p>characteristics of the investment fund market in Poland as an example of a market with high dynamics of changes in the functioning of open-end investment funds.</p>	<p>example of the market of open-end investment funds was done to prepare the ground for testing the model with the use of concrete data, which is a complementary to the simulations carried out. The legal basis and general principles of operation were considered, as well as the typology of investment funds and the dynamics of the development of the Polish market.</p>
<p>Construction of the quantitative model of the open-end investment fund operations.</p>	<p>The goal was realized in Chapter 4. The developed model is characterized by a holistic approach to modeling the FIO dynamics. Central equations are focused on the key variable: net asset value – It allows you to analyze, among others possibilities of a given fund, its popularity and stage of operation. Nevertheless, the model introduces all important processes to the description, embedding them in a discrete mathematical space composed of consecutive days of operation. Possible approximate situations are also considered (simplifying formulas and facilitating/enabling analysis) and description of the dynamics in any period of time using the general solution of a recursive equation.</p>
<p>Simulations and analyzes showing the scope of application of the constructed model.</p>	<p>The goal was realized in Chapter 5. After analyzing the scope of available data, characteristic for the Polish mutual fund market, the author focused on the basic aspects of functioning, that is, cash flows and the result described by the before-fee rate of return. In the first case, proof was also carried out showing that the proposed approach is a generalization of cash flow models we can find in the literature. At the end, the cost and FIO fees aspects were also analyzed.</p>

Source: own structure.

Methodology and results

The approach to construct the model proposed in this thesis belongs to the unique and still rarely considered in finances inductive approach to research, i.e. conducting research in the spirit of a machine learning experiment (data science), where the development of the proposed model is determined by real data, not research hypotheses. An integral part of this approach is the iterative work on the model, each time trying to improve its design.

The starting point of the model is to define “the operational algorithm/scenario of the day” of an open-end mutual fund: this scenario assumes the following operations: quantifying the amount of net capital flows and investment result as well as the calculation of operating fees, including management fees and performance fees. The author also considers two alternative day scenarios. After defining the scenarios, the author moves to describing the elements of these operations with mathematical formulas - from the fund's accumulation of initial capital, through determining the inflow and outflow of capital, determining the value of the participation unit (price) of a fund, to calculating the gross and net return after taking into account fixed and variable fund fees. He also uses generalizations and presents special cases, such as the size of fixed and variable costs, when the fund has losses or offers different categories of participation units. All formulas (there are 50 of them) are presented in detailed form with indices and descriptions and are numbered for transparency.

In the last stage of the study, the author simulates the accuracy of the model and the amount of measurement uncertainty in three designated areas, i.e. net flows, investment results and costs, using actual data from the Polish mutual fund market. He chooses this market because of its high development and many changes in legal and practical background that make it a perfect research trial. The data base consist of 500 open-end investment funds managed by 33 management companies between 2012 and 2019. The author presents the results of the simulations on the numerous illustrations The obtained results show that determining the values of the described three parameters on the basis of incomplete data may be a subject to high uncertainty, especially considering the high variability of their values in the analyzed period.

Main conclusions

The basic version of the proposed model accurately describes the actual functioning of an open-end investment fund and is modular in nature, i.e. the elements of the structure (expenses positions, cash flows, operating fees) are initially included additively and are included in the model according to the actual state of the art in each case. Thus, the base model can be adapted and applied to quantitatively describe any open-ended investment fund. Taking into account the specific aspects of the fund's operating activities allows the model to be used by practitioners employed either in an investment fund company or representing, for example, a transfer agent, in their daily work: (1) for analyzes and simulations of historical investment results, (2) for description future in the field of planning that performs management accounting tasks, (3) to prepare detailed business profitability analyzes for projects and scenarios undertaken in this area, allowing to study the sensitivity of the financial result to the parameters of designed solutions, (4) as well as for analyzes that allow you to compare your own products with competition (the so-called market intelligence). On the other hand, the work is theoretical - the model presents the operational side of the investment fund in a strictly quantitative manner and compatible with other areas of its activity. Therefore, the construction of specific layers of approximations in various analytical paths becomes important, as it allows the description to be adjusted to the commonly available data and to use the full information provided by these data. Thanks to this, depending on the selected issue, the model can be flexibly adapted and the funds can be examined at any chosen angle. It is worth mentioning that some specific elements of the structure are also taken into account, such as many categories of participation or the need to decompose flows and net asset value by distributors or investors (business partners) treated in particular.

As a result, the prepared model allows to search for the relationship between the issues related to the management of the portfolio of financial instruments and the obtained investment results, and the actual data available for observation for an unprivileged market participant, through the operational process of the investment fund's operations and their detailed analysis. This is particularly important in the ongoing period of dynamic regulatory changes regarding the cost side of investment funds in Poland. The finally prepared solutions also allow to a more accurate quantitative analysis of investors' behavior and, consequently,

for a more plausible qualitative description of these phenomena on the basis of behavioral sciences. However, these are some kind of "side effects", because the main result of the analyzes is a generalization and clarification of the connections between formulas and models commonly found in world literature.

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